☆ **Muller's Method** The muller method is used to improve the difficulty of Newton raphshon method. Newton Raphson raphson method & Bisection method are used to calculate the approximate coordinate of algebric eq^n but this method is used to determine the coordinate of non algebric eq^n.

Acc. to muller every non algebric eq^n represent its continuity for a certain derivative. It means that for higher order the non algebric eq^n remain discontinous for discontinuity he considered a fun^n ϵ s.t is represent the non convergent limit of non algebric eq^n. The mathematical form of muller method is given in the following manner.

→ Let x_i, x_{i-1} & x_{i-2} are three coordinates which represent an algebric eq^n. we also know that $y = f(x)$ then the coorresponding coordinates are y_i, y_{i-1}, y_{i-2} represent the func^n of algebric eq^n. Then two parameter A & B are define which give the rel^n b/w these coordinates & can be expressed as

$$P(x) = A(x - x_i)^2 + B(x - x_i) + y_i \quad \text{—(1)}$$

This is the eq^n when the curve is intersect at the point (x_i, y_i). If the intersection points are (x_{i-1}, y_{i-1}) & (x_{i-2}, y_{i-2}) Then above eq^n can be written as

$$y_{i-2} = A(x_{i-2} - x_i)^2 + B(x_{i-2} - x_i) + y_i \quad \text{—(2)}$$

$$\& \; y_{i-1} = A(x_{i-1} - x_i)^2 + B(x_{i-1} - x_i) + y_i \quad \text{—(3)}$$

These two eq^{n}s give the value of two parameters A & B.

Also from eq^n (2) & (3)

$$y_{i-2} - y_i = A(x_{i-2} - x_i)^2 + B(x_{i-2} - x_i) \quad \text{—(4)}$$

$$y_{i-1} - y_i = A(x_{i-1} - x_i)^2 + B(x_{i-1} - x_i) \quad \text{—(5)}$$

Now multiplying eq^n (4) by $(x_{i-1} - x_i)$ & eq^n (5) by $(x_{i-2} - x_i)$ & then substract we get

$$(y_{i-2} - y_i)(x_{i-1} - x_i) = A(x_{i-2} - x_i)^2 (x_{i-1} - x_i) + B(x_{i-2} - x_i)(x_{i-1} - x_i)$$

$$(y_{i-1} - y_i)(x_{i-2} - x_i) = A(x_{i-1} - x_i)^2 (x_{i-2} - x_i) + B(x_{i-1} - x_i)(x_{i-2} - x_i)$$

$$(y_{i-2} - y_i)(x_{i-1} - x_i) - (y_{i-1} - y_i)(x_{i-2} - x_i) = A[(x_{i-2} - x_i)(x_{i-1} - x_i)][(x_{i-2} - x_i) - x_{i-1} + x_i]$$

$$A = \frac{(y_{i-2}-y_i)(x_{i-1}-x_i)-(y_{i-1}-y_i)(x_{i-2}-x_i)}{(x_{i-2}-x_i)(x_{i-1}-x_i)(x_{i-2}-x_{i-1})}$$

$$A = \frac{(y_{i-2}-y_i)}{(x_{i-2}-x_i)(x_{i-2}-x_{i-1})} - \frac{(y_{i-1}-y_i)}{(x_{i-1}-x_i)(x_{i-2}-x_{i-1})}$$

also $$\boxed{A = \frac{y_{i-2}-y_i}{(x_{i-2}-x_i)(x_{i-1}-x_{i-2})} + \frac{(y_{i-2}-y_i)}{(x_{i-2}-x_i)(x_{i-2}-x_{i-1})}}$$

IIly $$\boxed{B = \frac{y_{i-1}-y_i}{(x_{i-1}-x_i)} - A(x_{i-1}-x_i)}$$

On putting these values in eq^n no (1) we get the next approximation whose value can be written as

$$\boxed{x_{i+1} = x_i - \frac{2y_i}{B \pm \sqrt{B^2-4Ay_i}}}$$

Here the value remain +ve when B is +ve & the value remain -ve when B is -ve.

Question If $y(x) = x^3-2x-5 = 0$ lies b/w 2 & 3 then calculate the roots of above eq^n?

Sol^n we have given $y(x) = x^3-2x-5$

$x_i = 3$, $x_{i-1} = 2$, $x_{i-2} = 1$

Then $y_i = (3)^3 - 2\times3 - 5 = 16$

$y_{i-1} = (2)^3 - 2\times2 - 5 = -1$

$y_{i-2} = (1)^3 - 2\times2 - 5 = -6$

Then the value of parameter A & B can be written as

$$A = \frac{-1-16}{(1-3)(2-1)} + \frac{-6-16}{(1-3)(1-2)}$$

$$= \frac{-17}{-1} - \frac{22}{2}$$

$$= 6$$

$$B = \frac{-1-16}{2-3} - 6(2-3)$$

$$= \frac{-17}{-1} - 6(-1) = 17+6 = 23$$

$$\rightarrow x_{i+1} = 3 - \frac{2\times16}{23 \pm \sqrt{(23)^2-4\times6\times16}}$$

$$= 3 - \frac{32}{23 \pm \sqrt{529-384}}$$

$$= 3 - \frac{32}{23+12.04}$$

$$= 2.09$$

$$\epsilon_i = \left|\left(\frac{x_{i+1}-x_i}{x_{i+1}}\right)\right| \times 100$$

$$\epsilon_i = \left|\left(\frac{2.086799548-3}{2.086}\right)\right| \times 100$$

$$\epsilon_i = 43.77\%$$

which is very higher Hence the next coordinate is to be calculated by considering the x_{i+1} as x_i

$x_i = 2.086799548$

$x_{i-1} = 2$ & $x_{i-2} = 1$

$y_{i-1} = -1$, $y_{i-2} = -6$

$y_i = (2.08)^3 - 2 \times 2.08 - 5$

$y_i = 8.98 - 4.32 - 5$

$y_i = 8.98 - 9.16$

$y_i = -0.17$

$$A = \frac{-1+0.17}{(2-2.08)(2-1)} + \frac{-6+0.17}{(1-2.08)(1-2)}$$

$$= \frac{-0.83}{-0.08\times 1} - \frac{5.83}{-1.08\times 1}$$

$$A = +\frac{0.83}{0.08} + \frac{5.83}{1.08}$$

$A = 10.37 - 5.3$

$A = 5.04$

$$B = \frac{-1+0.18}{1-2.08} - 5.08(2-2.08)$$

$$B = \frac{-0.82}{-1.08} - 5.08 \times -0.08$$

$B = 10.37 + 0.40$

$B = 10.96986338$

$x_{i+1} = 2.09462409$

$E_i = 0.3735535\ \%$

$$x_{i+1} = 2.08 - \frac{2 \times -0.17}{10.96 + \sqrt{120.12 - 4 \times 5.04 \times -}}$$

$$= 2.08 - \frac{0.34}{10.96 + 11.11}$$

$$= 2.08 - \frac{0.34}{22.07} = 2.08 - 0.015 = 2.1$$

$$E_i = \left| \frac{2.06 - 2.07}{2.06} \right| \times 100$$

Quotient difference method

This is the general method which is used to calculate the different (approximate) root of algebraic eq^n. Let a given algebraic eq^n is f(x)

$f(x) = a_0 x^3 + a_1 x^2 + a_2 x + a_3 = 0$

where a_0, a_1, a_2, a_3 are the different polynomials & let x_1, x_2, x_3 be the roots of this eq^n such that $0 < |x_1| < |x_2| < |x_3|$

then $f(x)$ can be written as

$$\frac{1}{f(x)} = \sum_{1} a_i x_i \quad \text{—— (1)}$$

Coulomb's Law for Continuous Charge Distributions

If a charge distribution is continuous, then the natural extension of Coulomb's Law is to *integrate* the electric field or force over the contributions from the infinitesimal charge elements dq at $\vec{r}\,'$:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o}\int \frac{1}{R^2}\,\widehat{R}\,dq(\vec{r}\,') \qquad (2.4)$$

where $\vec{R}$ varies with the location $\vec{r}\,'$ of dq as the integral is performed. dq is admittedly undefined here. However, before worrying about that, let us note that the integrand is a *vector* and so this integral requires some care: we must break up $\widehat{R}$ into its components and individually integrate each component. For example, if we use Cartesian coordinates, then $\widehat{R} = \widehat{x}\left(\widehat{R}\cdot\widehat{x}\right) + \widehat{y}\left(\widehat{R}\cdot\widehat{y}\right) + \widehat{z}\left(\widehat{R}\cdot\widehat{z}\right)$, and, since the Cartesian unit vectors do not depend on the location of the infinitesimal charge $dq(\vec{r}\,')$, we may write the integral out as follows:

$$\vec{E}(\vec{r}) = \qquad (2.5)$$
$$\frac{1}{4\pi\epsilon_o}\left[\widehat{x}\int\frac{1}{R^2}\left(\widehat{R}\cdot\widehat{x}\right)dq(\vec{r}\,') + \widehat{y}\int\frac{1}{R^2}\left(\widehat{R}\cdot\widehat{y}\right)dq(\vec{r}\,') + \widehat{z}\int\frac{1}{R^2}\left(\widehat{R}\cdot\widehat{z}\right)dq(\vec{r}\,')\right]$$

which is sum of three integrals with *scalar* integrands.

Now, consider some specific charge distributions:

- **volume charge distribution**:

$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o}\int_{\mathcal{V}}\frac{d\tau'\rho(\vec{r}\,')}{R^2}\,\widehat{R}$	with $\rho(\vec{r}\,')$ having units of C m^{-3}, $\vec{r}\,'$ running over all points in the volume distribution $\mathcal{V}$, and $d\tau'$ being the differential volume element at $\vec{r}\,'$ for $\mathcal{V}$	(2.6)

- **surface charge distribution**:

$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_o}\int_{\mathcal{S}}\frac{da'\sigma(\vec{r}\,')}{R^2}\,\widehat{R}$	with $\sigma(\vec{r}\,')$ having units of C m^{-2}, $\vec{r}\,'$ running over all points in the surface distribution $\mathcal{S}$, and da' being the differential area element at $\vec{r}\,'$ for $\mathcal{S}$	(2.7)

Coulomb's Law and the Electric Field

Coulomb's Law, Electrostatic Forces, and Superposition

We begin with two empirical facts:

- **Coulomb's Law**: the empirical fact that the force on a test charge q at position $\vec{r}$ due to a source charge q' at $\vec{r}'$ is given by Coulomb's Law:

$$\vec{F} = \frac{1}{4\,\pi\,\epsilon_o}\,\frac{q'\,q}{R^2}\,\widehat{R} \qquad \text{with} \quad \vec{R} \equiv \vec{r} - \vec{r}' \tag{2.1}$$

  where $\epsilon_o = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}. The force points along the line from q' to q as indicated by the sign of the definition of $\vec{R}$. The electric charge is in the units of *Coulombs* (C), which is a fundamental unit that cannot be written in terms of other fundamental units.

  Recall that: we use $\vec{}$ rather than boldface to indicate vectors; R where Griffiths uses a script r; and a different convention from Griffiths for the symbols for the two charges and their position vectors.

- **Superposition**: the empirical fact that Coulomb's Law obeys the principle of superposition: the force on a test charge q at $\vec{r}$ due to N charges $\{q_i'\}$ at positions $\{\vec{r}_i'\}$ is obtained by summing the individual vector forces:

$$\vec{F} = \sum_{i=1}^{N} \vec{F}_i = \sum_{i=1}^{N} \frac{1}{4\,\pi\,\epsilon_o}\,\frac{q_i'\,q}{R_i^2}\,\widehat{R}_i \qquad \text{with} \quad \vec{R}_i \equiv \vec{r} - \vec{r}_i' \tag{2.2}$$

The Electric Field

Given that any test charge q placed at the position $\vec{r}$ feels the same force, we are motivated to abstract away the test charge and define what we call the **electric field** at that position $\vec{r}$:

$$\boxed{\vec{E}(\vec{r}) = \frac{\vec{F}}{q} = \begin{cases} \frac{1}{4\,\pi\,\epsilon_o}\,\frac{q'}{R^2}\,\widehat{R} & \text{for a single source charge } q' \text{ at } \vec{r}' \\ \sum_{i=1}^{N} \frac{1}{4\,\pi\,\epsilon_o}\,\frac{q_i'}{R_i^2}\,\widehat{R}_i & \text{for } N \text{ source charges } \{q_i'\} \text{ at positions } \{\vec{r}_i'\} \end{cases}} \tag{2.3}$$

The electric field has units of N/C.

- **volume charge distribution**:

$$\vec{E}(\vec{r}) = \frac{1}{4\,\pi\,\epsilon_o}\int_{\mathcal{V}} \frac{d\tau'\rho(\vec{r}\,')}{R^2}\,\widehat{R}$$	with $\rho(\vec{r}\,')$ having units of C m^{-3}, $\vec{r}\,'$ running over all points in the volume distribution $\mathcal{V}$, and $d\tau'$ being the differential volume element at $\vec{r}\,'$ for $\mathcal{V}$	(2.6)

- **surface charge distribution**:

$$\vec{E}(\vec{r}) = \frac{1}{4\,\pi\,\epsilon_o}\int_{\mathcal{S}} \frac{da'\sigma(\vec{r}\,')}{R^2}\,\widehat{R}$$	with $\sigma(\vec{r}\,')$ having units of C m^{-2}, $\vec{r}\,'$ running over all points in the surface distribution $\mathcal{S}$, and da' being the differential area element at $\vec{r}\,'$ for $\mathcal{S}$	(2.7)

- **line charge distribution**:

$$\vec{E}(\vec{r}) = \frac{1}{4\,\pi\,\epsilon_o}\int_{\mathcal{C}} \frac{d\ell'\lambda(\vec{r}\,')}{R^2}\,\widehat{R}$$	with $\lambda(\vec{r}\,')$ having units of C m^{-1}, $\vec{r}\,'$ running over all points in the line distribution $\mathcal{C}$, and $d\ell'$ being the differential length element at $\vec{r}\,'$ for $\mathcal{C}$	(2.8)

Using the Dirac delta function we will define below, one can write the first two as special cases of the latter, using delta functions in the dimensions in which the charge distribution has no extent.